## Pure Mathematics 3

## Exercise 2A

1 a $\left|\frac{3}{4}\right|=\frac{3}{4}$
b $|-0.28|=0.28$
c $|3-11|=|-8|$

$$
=8
$$

d $\left|\frac{5}{7}-\frac{3}{8}\right|=\left|\frac{40}{56}-\frac{21}{56}\right|$

$$
=\frac{19}{56}
$$

e $\quad|20-6 \times 4|=|20-24|$

$$
\begin{aligned}
& =|-4| \\
& =4
\end{aligned}
$$

f $\left|4^{2} \times 2-3 \times 7\right|=|32-21|$

$$
=11
$$

2 a $f(1)=|7-5 \times 1|+3$

$$
\begin{aligned}
& =|7-5|+3 \\
& =5
\end{aligned}
$$

b $f(10)=|7-5 \times 10|+3$

$$
\begin{aligned}
& =|7-50|+3 \\
& =|-43|+3 \\
& =46
\end{aligned}
$$

c $\mathrm{f}(-6)=|7-5 \times(-6)|+3$
$=|7+30|+3$
$=40$
3 a $g(4)=\left|4^{2}-8 \times 4\right|$

$$
\begin{aligned}
& =|16-32| \\
& =|-16| \\
& =16
\end{aligned}
$$

3 b $\quad g(-5)=\left|(-5)^{2}-8 \times(-5)\right|$

$$
=|25+40|
$$

$$
=65
$$

c $g(8)=\left|8^{2}-8 \times 8\right|$

$$
=|64-64|
$$

$$
=0
$$

4 a


The graph meets the axes at $(1,0)$ and $(0,1)$
b


The graph meets the axes at $\left(-1 \frac{1}{2}, 0\right)$ and $(0,3)$

4 c


The graph meets the axes at
$\left(\frac{7}{4}, 0\right)$ and $(0,7)$
d


The graph meets the axes at $(10,0)$ and $(0,5)$
e


The graph meets the axes at $(7,0)$ and $(0,7)$

4 f


The graph meets the axes at $\left(\frac{3}{2}, 0\right)$ and $(0,6)$
g


The graph meets the axes at $(0,0)$
h


The graph meets the axes at $\left(\frac{1}{3}, 0\right)$ and $(0,-1)$

5 a

b At the left-hand point of intersection:

$$
\begin{aligned}
4-\frac{3}{2} x & =5 \\
\frac{3}{2} x & =-1 \\
x & =-\frac{2}{3}
\end{aligned}
$$

At the right-hand point of intersection:

$$
\begin{aligned}
-\left(4-\frac{3}{2} x\right) & =5 \\
\frac{3}{2} x & =9 \\
x & =6
\end{aligned}
$$

The solutions are $x=-\frac{2}{3}$ and $x=6$

6 a


At A: $-(3 x-1)=5$
$-3 x=4$

$$
x=-\frac{4}{3}
$$

At B: $3 x-1=5$

$$
\begin{aligned}
3 x & =6 \\
x & =2
\end{aligned}
$$

The solutions are $x=-\frac{4}{3}$ and $x=2$
b


At A: $\begin{aligned}-\left(\frac{x-5}{2}\right) & =1 \\ x-5 & =-2 \\ x & =3\end{aligned}$
At B: $\frac{x-5}{2}=1$

$$
x-5=2
$$

$$
x=7
$$

The solutions are $x=3$ and $x=7$

6 c


The graphs do not intersect so there are no solutions.
d


At A: $-(7 x-3)=4$

$$
\begin{aligned}
7 x & =-1 \\
x & =-\frac{1}{7}
\end{aligned}
$$

At B: $7 x-3=4$

$$
\begin{aligned}
7 x & =7 \\
x & =1
\end{aligned}
$$

The solutions are $x=-\frac{1}{7}$ and $x=1$
At B. $7 x-3=4$-

6 e


$$
\text { At } \begin{aligned}
\mathrm{A}: \frac{4-5 x}{3} & =2 \\
-5 x & =2 \\
x & =-\frac{2}{5}
\end{aligned}
$$

At B: $-\left(\frac{4-5 x}{3}\right)=2$

$$
-5 x=-10
$$

$$
x=2
$$

The solutions are $x=-\frac{2}{5}$ and $x=2$

6 f


At A: $-\left(\frac{x}{6}-1\right)=3$

$$
\begin{aligned}
\frac{x}{6} & =-2 \\
x & =-12
\end{aligned}
$$

At B: $\frac{x}{6}-1=3$

$$
\begin{aligned}
& \frac{x}{6}=4 \\
& x=24
\end{aligned}
$$

The solutions are

$$
x=-12 \text { and } x=24
$$

7 a


7 b Intersection point $A$ is on the reflected part of $y=\frac{1}{2} x-2$

$$
\begin{aligned}
-\left(\frac{1}{2} x-2\right) & =-2 x \\
2 x-\frac{1}{2} x & =-2 \\
\frac{3}{2} x & =-2
\end{aligned}
$$

$$
x=-\frac{4}{3}
$$

8


$$
\text { At A: } \begin{aligned}
-(3 x-5) & =11-x \\
-6 & =2 x \\
x & =-3
\end{aligned}
$$

At B: $3 x-5=11-x$

$$
\begin{aligned}
4 x & =16 \\
x & =4
\end{aligned}
$$

The solutions are $x=-3$ and $x=4$

9 a

b The two graphs do not intersect, therefore there are no solutions to the equation $|6-x|=\frac{1}{2} x-5$

10 The value for $x$ cannot be negative as it equals a modulus which is $\geq 0$

## 11 a


b At the left-hand point of intersection:

$$
\begin{aligned}
3 x+4 & =2 x-9 \\
x & =-13
\end{aligned}
$$

At the right-hand point of intersection:

$$
\begin{aligned}
-(3 x+4) & =2 x-9 \\
-5 x & =-5 \\
x & =1
\end{aligned}
$$

The points of intersection are $x=-13$ and $x=1$

So the solution to $-|3 x+4|<2 x-9$ is $x<-13$ and $x>1$

12


At A: $-(2 x+9)=14-x$

$$
-x=23
$$

$$
x=-23
$$

At B: $2 x+9=14-x$

$$
\begin{aligned}
3 x & =5 \\
x & =\frac{5}{3}
\end{aligned}
$$

The points of intersection are
$x=-23$ and $x=\frac{5}{3}$
So the solution to $|2 x+9|<14-x$
is $-23<x<\frac{5}{3}$

13 a For there to be one solution, the
graphs $y=|6-x|$ and $y=\frac{1}{2} x+k$
must intersect once at the vertex of $y=|6-x|$


This vertex occurs at $(6,0)$
Substituting $(6,0)$ into $y=\frac{1}{2} x+k$
gives:
$0=\frac{1}{2} \times 6+k$
$0=3+k$
$k=-3$
b $6-x=\frac{1}{2} x-3$

$$
9=\frac{3}{2} x
$$

$$
x=6
$$

## Challenge

a

b At the far left-hand and far right-hand points of intersection:
$x^{2}+9 x+8=1-x$
$x^{2}+10 x+7=0$
Using the formula:
$x=\frac{-10 \pm \sqrt{10^{2}-4 \times 1 \times 7}}{2 \times 1}$
$x=\frac{-10 \pm \sqrt{72}}{2}$
$x=\frac{-10 \pm 6 \sqrt{2}}{2}$
$x=-5 \pm 3 \sqrt{2}$
At the two inside points of intersection:

$$
\begin{aligned}
-\left(x^{2}+9 x+8\right) & =1-x \\
x^{2}+9 x+8 & =x-1 \\
x^{2}+8 x+9 & =0
\end{aligned}
$$

Using the formula:
$x=\frac{-8 \pm \sqrt{8^{2}-4 \times 1 \times 9}}{2 \times 1}$
$x=\frac{-8 \pm \sqrt{28}}{2}$
$x=\frac{-8 \pm 2 \sqrt{7}}{2}$
$x=-4 \pm \sqrt{7}$
The four solutions are $x=-5 \pm 3 \sqrt{2}$ and $x=-4 \pm \sqrt{7}$

